

# New Constraints on “Cool” Dark Matter

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## Abstract

It has been suggested that a sterile neutrino  $\nu_s$  which mixes with standard neutrinos can form nonthermal “cool” Dark Matter if its mass and mixing angle fall in the ranges  $0.1 \text{ keV} \lesssim m_s \lesssim 10 \text{ keV}$  and  $10^{-10} \lesssim \sin^2 \theta \lesssim 10^{-4}$ , respectively. We point out that the required mixing makes these heavy neutrinos unstable. The dominant decay mode is into three light neutrinos, but the most stringent constraint comes from the non-observation of radiative decays into a single light neutrino and a photon. Moreover, we point out that the density of thermal relics of such  $\nu_s$ s would be too high, unless the reheating temperature after inflation was below  $\sim 10 \text{ GeV}$ .

Recently Shi and Fuller suggested [1] a new particle physics candidate for the Dark Matter in the Universe. They note that, if a significant lepton-antilepton asymmetry  $\Delta L$  existed in the early Universe and was reflected in a net neutrino number, and if there is a sterile neutrino species  $\nu_s$  with an appropriate mass and mixing angles, the standard neutrino excess is converted into a sterile neutrino excess via resonant interactions with the thermal plasma at a temperature of  $\sim 10$  MeV. Since neutrinos with the lowest energy will be converted first, until  $\Delta L$  is essentially used up, the resulting sterile neutrinos, while mostly relativistic, will be significantly “colder” than the standard neutrinos. More exactly, their energy distribution will be nonthermal, with average energy a fraction of that of the parent (standard) neutrinos. This nonthermal ensemble of sterile neutrinos  $\nu_s$  would survive to this day, and constitutes a suitable Dark Matter candidate for mass and mixing angle parameters in the ranges  $0.1 \text{ keV} \lesssim m_s \lesssim 10 \text{ keV}$  and  $10^{-10} \lesssim \sin^2 \theta \lesssim 10^{-4}$ , respectively. Such “cool” Dark Matter might be compatible with studies of structure formation, while hot Dark Matter seems to be excluded. An explicit realization of this idea, where  $\nu_s$  is a supersymmetric axino, has been suggested by Chun and Kim [2].

The authors of refs. [1, 2] seem to have overlooked the fact that the mixing of  $\nu_s$  with standard neutrinos will cause the  $\nu_s$  to decay, and will induce interactions between  $\nu_s$  and the thermal plasma. Obviously,  $\nu_s$  can only form Dark Matter if it is sufficiently long-lived. In addition, it will only form *cool* Dark Matter if the density of thermal  $\nu_s$  relics is sufficiently small. We find that the first condition imposes a significant constraint on parameter space that is independent of the thermal history of the Universe. The second condition excludes the model altogether, if the post-inflationary Universe was ever hot enough for  $\nu_s$  to have been in thermal equilibrium. Even for the smallest mixing angle  $\sin^2 \theta \sim 10^{-10}$  compatible with efficient conversion of a lepton asymmetry  $\Delta L$  into  $\nu_s$ s, the  $\nu_s$  freeze-out temperature only amounts to  $\sim 10$  GeV. “Cool” Dark Matter would thus require the reheating temperature after inflation to be below this value.

We start our discussion with an analysis of  $\nu_s$  decays. Since we are interested in sterile neutrinos with mass  $m_s \lesssim 10 \text{ keV}$ , the only possible tree-level decay is the one into three light Standard Model (SM) neutrinos,  $\nu_s \rightarrow 3\nu$ , which proceeds through the exchange of a  $Z$ -boson. The  $Z\nu_s\bar{\nu}$  coupling is simply  $\sin \theta$  times the  $Z\nu\bar{\nu}$  coupling of the SM, where  $\theta$  is the  $\nu - \nu_s$  mixing angle. The corresponding partial width is given by

$$\Gamma(\nu_s \rightarrow 3\nu) = \frac{G_F^2 m_s^5}{192\pi^3} \sin^2 \theta, \quad (1)$$

where  $G_F$  is the Fermi constant, and we have summed over all three generations of SM neutrinos.  $\sin^2 \theta = \sum_i \sin^2 \theta_i$ , where  $\theta_i$  is the mixing angle between  $\nu_s$  and  $\nu_i$ , is the effective mixing angle to standard model neutrinos.

An obvious constraint on any particle physics candidate for Dark Matter is that its lifetime should exceed the age of the Universe,  $\tau(\nu_s) \gtrsim 5 \cdot 10^{17} \text{ sec}$ . However, in the case at hand a stronger constraint can be derived, since the mixing between  $\nu_s$  and the light neutrinos also induces radiative decays at the one-loop level, with branching ratio [3]

$$B(\nu_s \rightarrow \nu\gamma) = \frac{27\alpha_{\text{em}}}{8\pi} \simeq 8 \cdot 10^{-3}, \quad (2)$$

where  $\alpha_{\text{em}}$  is the fine structure constant. These decays would add a monochromatic line at energy  $E = m_s/2$  to the diffuse background of hard UV or soft X-ray photons. There

are stringent observational bounds on such anomalies [4], leading very conservatively to the requirement\*

$$\tau(\nu_s) > 10^{22} \text{ sec.} \quad (3)$$

This implies

$$\sin^2 \theta < 2.9 \cdot 10^{-3} \left( \frac{1 \text{ keV}}{m_s} \right)^5, \quad (4)$$

where we have used eq. (1). This condition imposes a nontrivial new constraint for  $m_s \gtrsim 2$  keV. Note that this constraint does not depend on the thermal history of the Universe prior of the conversion of the lepton asymmetry into sterile neutrinos. It is depicted by the solid line in Fig. 1.

The constraint (3) has been derived [4] under the conservative assumption that Dark Matter is distributed uniformly throughout the Universe. A simple estimate shows that the contribution from the dark halo of our galaxy alone yields a comparable bound. The best strategy would probably be to search for the emission of monoenergetic photons from regions that are known to be rich in Dark Matter, but have few background sources in the relevant frequency band. One example might be (the centers of) dwarf galaxies.

Of course, these searches can only be expected to be successful if  $\nu_s$  does indeed form (most of) the Dark Matter in the Universe. We will now show that the requirement of thermal nonequilibrium, the second condition mentioned above, imposes a severe constraint on the thermal history of the Universe if Dark Matter is indeed “cool”. This constraint follows from the requirement that the present density of *thermal* relic  $\nu_s$ s, which contribute to hot Dark Matter, should be sufficiently small. Since  $\nu_s$  was relativistic when it decoupled its contribution to the present mass density of the Universe can be obtained by simple scaling from the contribution of massive SM neutrinos [5, 4]:

$$\Omega_s^{\text{thermal}} h^2 \simeq \frac{m_s}{100 \text{ eV}} \cdot \frac{10.75}{g_*(T_F)}, \quad (5)$$

where  $h$  is today’s Hubble constant in units of 100 km/(sec·Mpc), and  $g_*$  is the number of relativistic degrees of freedom at the temperature  $T_F$  where  $\nu_s$  decoupled from the plasma of SM particles. This temperature is defined by the condition that the rate of reactions that change the number density  $n_s$  of  $\nu_s$  should be equal to the Hubble expansion rate  $H$  at that temperature:

$$n_s(T_F) \langle v \sigma(\nu_s f \rightarrow \nu f) \rangle(T_F) = H(T_F). \quad (6)$$

Here  $f$  stands for any SM fermion or antifermion and  $\nu$  for an active neutrino. The symbol  $\langle \dots \rangle$  denotes the thermal average.

During the radiation dominated era, the Hubble expansion rate was [4]

$$H(T) = \frac{\pi T^2}{M_{Pl}} \sqrt{\frac{g_*}{90}}, \quad (7)$$

where  $M_{Pl} = 2.4 \cdot 10^{18}$  GeV is the reduced Planck mass. Since by assumption  $\nu_s$  was in equilibrium for  $T \geq T_F$ ,  $n_s$  in eq. (6) can be replaced by the equilibrium density [4]

$$n_s = n_s^{\text{eq}} = \frac{3\xi(3)}{2\pi^2} T^3 = 0.183 T^3. \quad (8)$$

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\*This constraint is weaker than the corresponding one in ref. [4] by a factor of the branching ratio (2). Note also that we assume  $\nu_s$  to form all Dark Matter, independently of  $m_s$ ; this can be arranged by an appropriate choice of  $\Delta L$ . As a result, our limit (3) is independent of  $m_s$ , since the neutrino density  $n_s$  and the observational upper bound on the photon flux both scale like  $1/m_s$ .

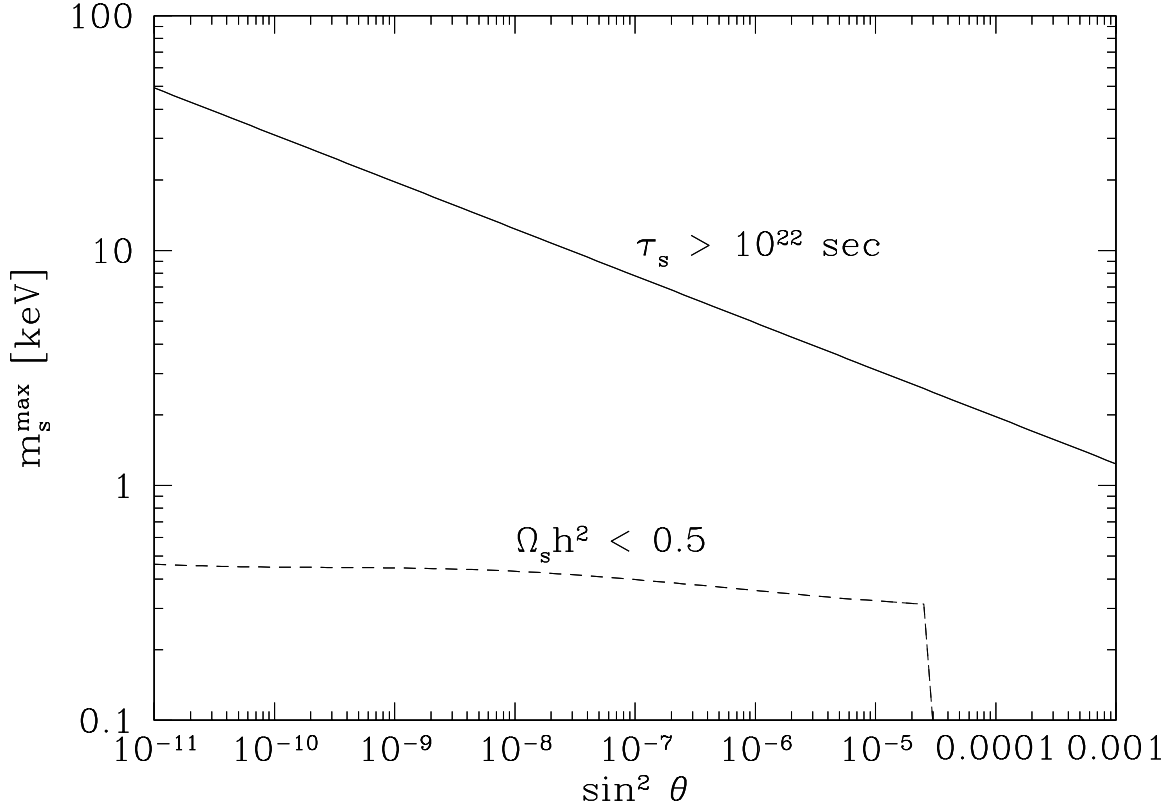


Figure 1: Upper bounds on the mass  $m_s$  of the sterile neutrino  $\nu_s$  as a function of  $\sin^2 \theta$ ,  $\theta$  being the effective mixing angle to SM neutrinos. The solid line comes from the upper bound on radiative  $\nu_s$  decays and is independent of the thermal history of the Universe. The dashed line follows from the requirement that *thermal*  $\nu_s$  relics should not “overclose” the Universe; it has been derived under the assumption that  $\nu_s$  was in thermal equilibrium after inflation.

Here we have conservatively assumed that  $\nu_s$  effectively has only two degrees of freedom (left-handed  $\nu_s$  and right-handed  $\bar{\nu}_s$ ); if  $\nu_s$  is a Dirac particle, this amounts to the assumption that interactions which change the chirality of  $\nu_s$  are not in equilibrium at  $T \simeq T_F$ .

We estimate the cross section appearing in eq. (6) from the  $\nu_\mu e^- \rightarrow \nu_\mu e^-$  scattering cross section. This is again conservative, since it ignores charged current contributions to the cross section; moreover, electrons have the smallest (vector) couplings to the  $Z$  among all SM fermions. This gives

$$\langle \nu \sigma(\nu_s f \rightarrow \nu f) \rangle = \sin^2 \theta \cdot \frac{G_F^2}{12\pi} (3 - 12 \sin^2 \theta_W + 16 \sin^4 \theta_W) \cdot \langle s \rangle, \quad (9)$$

where  $\theta_W$  is the weak mixing angle. We have ignored the momentum dependence of the  $Z$  propagator, since  $T_F^2 \ll M_Z^2$  for all cases of interest. Moreover, when computing the thermal average, we ignore the Fermi blocking in the final state. The thermal average of the squared center-of-mass energy  $s$  is then given by

$$\langle s \rangle = 2 \langle E_1 E_2 \rangle = 2T^2 \left[ \frac{\int_0^\infty dx x^3 / (1 + e^x)}{\int_0^\infty dx x^2 / (1 + e^x)} \right]^2 = 19.8 T^2, \quad (10)$$

where  $E_1$  and  $E_2$  are the energies of the two particles in the initial state in the co-moving frame. Putting everything together, we have

$$T_F \simeq 2.2 \text{ MeV} \cdot (\sin^2 \theta)^{-1/3} \cdot [g_*(T_F)]^{1/6}. \quad (11)$$

The factor  $[g_*(T_F)]^{1/6}$  varies between 1.5 for  $T_F \sim 10 \text{ MeV}$  and 2.1 for  $T_F \sim 10 \text{ GeV}$ . Note that  $T_F \lesssim 10 \text{ GeV}$  for  $\sin^2 \theta \gtrsim 10^{-10}$ .

The dashed line in Fig. 1 shows the upper bound on  $m_s$  that follows from eqs.(5) and (11) by requiring that  $\nu_s$  should not “overclose” the Universe,  $\Omega_s^{\text{thermal}} h^2 < 0.5$ . The drop at  $\sin^2 \theta = 3 \cdot 10^{-5}$  occurs since for larger mixing angles  $T_F$  falls below the temperature of the QCD phase transition, which we conservatively assumed to be  $150 \text{ MeV}$ ; this leads to a significant decrease of  $g_*(T_F)$  in eq. (5). Note that the upper bound on  $m_s$  should be lowered by at least another factor of five if  $\nu_s$  is to form mostly *cool* Dark Matter, i. e. if thermal (hot) relics are to form only a minor fraction of today’s  $\nu_s$  relic density. The *upper* bound on  $m_s$  that follows from the upper bound on the density of thermal  $\nu_s$  relics would then fall slightly *below* the *lower* bound on  $m_s$  of  $\sim 0.1 \text{ keV}$  derived in ref. [1].

Simply put, “cool” Dark Matter as discussed in refs. [1, 2] cannot exist if  $\nu_s$  ever was in thermal equilibrium. The “cool” Dark Matter model is therefore only viable if the reheating temperature  $T_R$  after inflation is (well) below the temperature (11). For example, certain models of  $D$ -term inflation have  $T_R \sim 1 \text{ GeV}$  [6]; this would leave some parameter space with  $\sin^2 \theta < 10^{-7}$ . Note, however, that the density of thermally produced  $\nu_s$  can be significant even if  $\nu_s$  never was in thermal equilibrium. This is analogous to the case of gravitinos with mass  $\gtrsim 1 \text{ keV}$  in supergravity models. For a given  $T_R < T_F$  and a given  $\sin^2 \theta$ , a nontrivial upper bound on  $m_s$  may therefore still result. Of course, the requirement of a very low reheating temperature may also make it difficult to explain the large lepton asymmetry that is required by the model.

In summary, we have shown that the requirement that the sterile neutrino be sufficiently long-lived imposes a significant constraint on the allowed combinations of mass  $m_s$  and mixing angle  $\theta$  with ordinary (active) neutrinos. Loop induced radiative  $\nu_s \rightarrow \nu \gamma$  decays play a crucial role here. This constraint is independent of the thermal history of the Universe. Furthermore, the  $\nu_s$  can only form *cool* Dark Matter if, after the end of inflation, it never was in thermal equilibrium. This imposes a very stringent constraint on the reheating temperature; e. g.,  $T_R < 10 \text{ GeV}$  ( $1 \text{ GeV}$ ) for  $\sin^2 \theta = 10^{-10}$  ( $10^{-7}$ ). These constraints, together with the requirement of a lepton asymmetry that is several orders of magnitude larger than the observed baryon asymmetry, make this model rather unattractive.

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